Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

the author has shown, suggests a method of determining the molecular arrangement of an alloy; and, in general, whether any alloy be a chemical compound or a mixture.

The author also enters into several details as to peculiar, and, in some cases, singular reaction of these and other alloys upon solutions of the salts of their own metals: thus, certain alloys of lead and zinc decompose solutions of lead as rapidly as pure zinc; while others, containing much zinc, act as lead towards the salts of lead.

In the case of three metals, A, B, C, whereof A is $\varepsilon +$, and C is $\varepsilon -$ to B, the author investigates the question as to what will be the electro-chemical relation of the atomic alloys of $A_x + C_y$ towards B, in solvent menstrua; and in the class of alloys of copper and zinc, has determined *the alloy of no action*, with reference to iron; and has also found alloys which protect iron in solvents electro-chemically as fully as pure zinc, and yet are not themselves acted on by the solvent.

He enters into the subject of the specific gravities of the alloys of Zn + Cu and Sn + Cu minutely, and shows reason to doubt the accuracy of the published specific gravities of most alloys of these and some other classes.

Professor Mac Cullagh read a supplement to his paper "On the dynamical Theory of Crystalline Reflexion and Refraction."

In his former paper on that subject (see Proceedings, 9th December, 1839) the author had given the general principles for solving all questions relative to the propagation of light in a given medium, or its reflexion and refraction at the separating surface of two media; but he had applied them only to the common case of waves, which suffer no diminution of intensity in their progress, and in which the vibration may be represented by the sine or cosine of an arc multiplied by a constant quantity. Some months after that paper was read, it occurred to him that he might obtain new and important results by substituting in his differential equations of motion a more general expression for the integral, that is, (as usual in such problems), by making the displacements proportional to the sine or cosine of an arc, multiplied by a negative exponential, of which the exponent should be a linear function of the coordinates. Such vibrations would become very rapidly insensible, and would, therefore, be fitted to represent the disturbance which, in the case of *total reflexion*, takes place immediately behind the reflecting surface; and the laws of this disturbance being thus discovered, the laws of polarization in the totally reflected light would also become known, by means of the general formulæ which the author had established for all cases of reflexion at the common surface of two media.

The present supplement is the fruit of these considerations. It contains the complete theory of the new kind of vibrations, not only in ordinary media, but in doubly refracting crystals; and also the complete discussion of the laws of total reflexion at the first or second surface of a crystal, including, as a particular case, the well known empirical formulæ of Fresnel for total reflexion at the surface of an ordinary medium.

The existence of vibrations represented by an expression containing a negative exponential as a factor, had been recognized by other writers, and was indeed sufficiently indicated by the phenomenon of total reflexion; but it was impossible to obtain the laws of such vibrations, so long as the general equations for the propagation of light were unknown.

The method of deducing these equations was given in the abstract of the author's former paper, (see Proceedings, as above); but as they were not there stated, it may be well to transcribe them here. If then we put

$$\text{X} = \frac{d\eta}{dz} - \frac{d\zeta}{dy}, \quad \text{Y} = \frac{d\zeta}{dx} - \frac{d\xi}{dz}, \quad \text{Z} = \frac{d\xi}{dy} - \frac{d\eta}{dx}, \qquad (1)$$

and suppose the axes of coordinates to be the principal axes of the crystal, the equations, in question may be thus written:

$$\left.\begin{aligned} \frac{d^2\xi}{dt^2} &= c^2\frac{d\mathrm{Z}}{dy} - b^2\frac{d\mathrm{Y}}{dz}, \\ \frac{d^2\eta}{dt^2} &= a^2\frac{d\mathrm{X}}{dz} - c^2\frac{d\mathrm{Z}}{dx}, \\ \frac{d^2\zeta}{dt^2} &= b^2\frac{d\mathrm{Y}}{dx} - a^2\frac{d\mathrm{X}}{dy}; \end{aligned}\right\} \quad (2)$$

and if we further put

$$\xi = \frac{d\eta_1}{dz} - \frac{d\zeta_1}{dy}, \quad \eta = \frac{d\zeta_1}{dx} - \frac{d\xi_1}{dz}, \quad \zeta = \frac{d\xi_1}{dy} - \frac{d\eta_1}{dx}, \quad (3)$$

they will take the following simple form:

$$\frac{d^2\xi_1}{dt^2} = -a^2\mathrm{X}, \quad \frac{d^2\eta_1}{dt^2} = -b^2\mathrm{Y}, \quad \frac{d^2\zeta_1}{dt^2} = -c^2\mathrm{Z}, \quad (4)$$

in which it is remarkable that the auxiliary quantities ξ_1, η_1, ζ_1, are exactly, for an ordinary medium, the components of the displacement in the theory of Fresnel. In a doubly defracting crystal, the resultant of ξ_1, η_1, ζ_1 is perpendicular to the *ray*, and comprised in a plane passing through the ray and the wave normal. Its amplitude, or greatest magnitude, is proportional to the amplitude of the vibration itself, multiplied by the velocity of the ray.

The conditions to be fulfilled at the separating surface of two media were given in the abstract already referred to. From these it follows, that the resultant of the quantities ξ_1, η_1, ζ_1, projected on that surface, is the same in both media; but the part perpendicular to the surface is not the same; whereas the quantities ξ, η, ζ, are identical in both. These assertions, analytically expressed, would give five equations, though four are sufficient; but it can be shown that any one of the equations is implied in the other four, not only in the case of common, but of total reflexion; which is a very remarkable circumstance, and a very strong confirmation of the theory.

The laws of double refraction, discovered by Fresnel, but not legitimately deduced from a consistent hypothesis, either by himself or any intermediate writer, may be very easily obtained, as the author has already shown, from equations (2), by assuming

$$\xi = p \cos \alpha \sin \phi, \quad \eta = p \cos \beta \sin \phi, \quad \zeta = p \cos \gamma \sin \phi, \tag{5}$$

where

$$\phi = \frac{2\pi}{\lambda} (lx + my + nz - st);$$

but the new laws, which are the object of the present supplement, are to be obtained from the same equations by making

$$\left.\begin{aligned} \xi &= \varepsilon (p \cos \alpha \sin \phi + q \cos \alpha' \cos \phi), \\ \eta &= \varepsilon (p \cos \beta \sin \phi + q \cos \beta' \cos \phi), \\ \zeta &= \varepsilon (p \cos \gamma \sin \phi + q \cos \gamma' \cos \phi), \end{aligned}\right\} \tag{6}$$

where ϕ has the same signification as before, and

$$\varepsilon = e^{-\frac{2\pi r}{\lambda}(fx + gy + hz)};$$

the vibrations being now elliptical, whereas in the former case they were rectilinear. In these elliptic vibrations the motion depends not only on the distance of the vibrating particle from the plane whose equation is

$$lx + my + nz = 0, \tag{7}$$

but also on its distance from the plane expressed by the equation

$$fx + gy + hz = 0; \tag{8}$$

and if the constants in the equation of each plane denote the cosines of the angles which it makes with the coordinate planes, we shall have λ for the length of the wave, and s for the velocity of propagation; while the rapidity with which the motion is extinguished, in receding from the second plane, will depend upon the constant r. The constants p and q may be any two conjugate semidiameters of the ellipse in which the vibration is performed; the former making, with the axes of coordinates, the angles α, β, γ, the latter the angles α', β', γ'.

As vibrations of this kind cannot exist in any medium, unless they are maintained by total reflexion at its surface, we shall suppose, in order to contemplate their laws in their utmost generality, that a crystal is in contact with a fluid of greater refractive power than itself, and that a ray is incident at their common surface, at such an angle as to produce total reflexion. The question then is, the angle of incidence being given, to determine the laws of the disturbance within the crystal.

The author finds that the refraction is still *double*, and that two distinct and separable systems of vibration are transmitted into the crystal. He shows that the surface of the crystal itself (the origin of coordinates being upon it at the point of incidence) must coincide with the plane expressed by equation (8), a circumstance which determines the three constants f, g, h. The plane expressed by (7) is parallel to the plane of the refracted wave; and a normal, drawn to it through the origin, lies in the plane of incidence, making with a perpendicular to the face of the crystal an angle ω which may be called the angle of refraction, so that, if i be the angle of incidence, we have

$$\sin \omega = s \sin i,$$

the velocity of propagation in the fluid being regarded as unity.

To each refracted wave, or system of vibration, corresponds a particular system of values for r, s, ω. These the author shows how to determine by means of the *index-surface* (the reciprocal of Fresnel's wave surface) which he has employed on other occasions, (Transactions of the Academy, vol. xvii. and xviii.), and the rule which he gives for this purpose affords a remarkable example of the use of the imaginary roots of equations, without the theory of which, indeed, it would have been difficult to prove, in the present instance, that there are two, and only two, refracted waves. Taking a new system of coordinates x', y', z', of which z' is perpendicular to the surface of the crystal, and y' to the plane of incidence, while x' lies in the intersection of these two planes, put $y'=0$ in the equation of the index-surface referred to those coordinates, the origin being at its centre; we shall then have an equation of the fourth degree between x' and z', which will be the equation of the section made in the index surface by the plane of incidence. In this equation put $x' = \sin i$, and then solve it for z'. When i exceeds a certain angle i', the four values of z' will be imaginary, and if they be denoted by

$$u \pm v\sqrt{-1}, \qquad u' \pm v'\sqrt{-1},$$

each pair will correspond to a refracted system, and we shall have, for the first,

$$\tan\omega = \frac{\sin i}{u}, \qquad s = \frac{\sin\omega}{\sin i}, \qquad r = sv; \qquad (9)$$

and for the second,

$$\tan\omega' = \frac{\sin i}{u'}, \qquad s' = \frac{\sin\omega'}{\sin i}, \qquad r' = s'v'. \qquad (10)$$

When i lies between i' and a certain smaller angle i'', two of the roots will be real, and two imaginary. The real roots correspond to waves which follow the law of Fresnel; the imaginary roots give a single wave, following the other laws just mentioned.

Lastly, when i is less than i'', all the roots are real, the refraction is entirely regulated by Fresnel's law, and the reflexion by the laws already discovered and published by the author.

If the crystal be uniaxal, and all the values of z' imaginary, the ordinary wave normal will coincide with the axis of x'; whilst the extraordinary wave normal and the axis of z' will be conjugate diameters of the ellipse in which the index surface is cut by the plane of incidence.

When $a = b = c$, the crystal becomes an ordinary medium; there is then only single refraction, and the refracted wave is always perpendicular to the axis of x'.

With regard to the ellipse in which the vibrations are performed, it may be worth while to observe, that if it be projected perpendicularly on the plane of incidence, the projected diameters which are parallel to the surface of the crystal and to the wave plane will, in all cases, be conjugate to each other, and their respective lengths will be in the proportion of r to unity. The vibrations, it is obvious, are not performed in the plane of the wave, though they take place without changing the density of the ether.

The new laws here announced are, properly speaking, laws of double refraction, and are necessary to complete our knowledge of that subject. Between them and the laws of Fresnel a curious analogy exists, founded on the change of real into imaginary constants.

The laws of the total reflexion, which accompanies the new kind of refraction, need not be dwelt upon in this abstract, as nothing is now more easy than to form the equations which contain them. In fact, the difficulties which formerly surrounded the problem of reflexion, even in the simplest cases, have completely disappeared, since the author made known the conditions which must be fulfilled at the separating surface of two media.

In what precedes, it has been supposed that the reflexion and refraction take place at the *first* surface of the crystal, because this is the more difficult and complicated of the two cases into which the question resolves itself. But it will usually happen in practice that a ray which has entered the crystal will suffer total reflexion at the *second* surface, while the new kind of vibration is propagated into the air without. The refracted wave will then be always perpendicular to the axis of x'; the two reflected rays, within the crystal, will be plane-polarized, according to the common law, but they will each undergo a change of phase; and the *vis viva* of the two rays together will be equal to that of the incident ray, the *vis viva* being measured by the square of the amplitude multiplied by the proportional mass.

In conclusion, the author states a mathematical hypothesis by which both the laws of dispersion, and those of the elliptic polarization of rock crystal, may be connected with the laws already developed.

The Rev. Dr. Todd communicated the following particulars concerning an ancient inkstand, contained in a letter from Joseph H. Monck Mason, Esq., dated Rome, May 4th, 1841:

" This relic (described by Mrs. Hamilton Gray, in her Tour in Etruria, p. 334) is of coarse *terra-cotta*, black and ill-formed; it is a truncated cone, about eight inches long, about four or five inches *round* the mouth, and about twelve (not more, I think) round the base; there are about twelve lines of letters written round the upper three-fourths, and one round the very base; they are written from left to right. I repeat it, that, being cramped by the rules and restrictions, I was not entirely particular; and the upper part was dirty, and not very legible without close inspection. I could see of the last line (I do not mean that round the base, which has some Greek capitals,) about three-fifths; this contained fourteen letters. I saw no *omicron* or *omega*. This line might have had twenty-four letters; and allowing about twelve, or say fifteen lines in all, gradually diminishing as on the outside of a cone, there was scarcely room for *ba*, *be*, in a second language. I believe them all to be Greek. About three or four lines up, I remember, there was P (*ro*,) and